Modélisation et identification des caractéristiques d'une structure vibratoire:un problème de réalisation stochastique d'un grand système non stationnaire

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MODELISATION ET IDENTIFICATION DES CARACTERISTIQUES D'UNE STRUCTURE VIBRATOIRE : UN PROBLEME DE REALISATION STOCHASTIQUE D'UN GRAND SYSTEME NON STATIONNAIRE

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Résumé : La modélisation et l'identification des modes de vibration d'une structure soumise à une excitation naturelle peuvent être considérées comme un problème de réalisation stochastique. Vu sous cet angle, ce problème présente les difficultés suivantes :

1) — l'excitation est non stationnaire,
2) — il s'agit de grands systèmes car on souhaite identifier un grand nombre de modes afin de détecter l'apparition éventuelle de fatigues dans la structure. On donne des résultats théoriques et expérimentaux pour ce problème.

Abstract : The modelling and identification of vibrating structures subject to natural excitation is considered in this paper as a stochastic realization problem. The main difficulties are the following : the excitation is nonstationary, and the system is of large scale since a large amount of poles and modes have to be identified for the purpose of detecting fatigues in the vibrating structure. Theoretical and experimental results are presented for this problem.

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I - INTRODUCTION

Vibration monitoring is a basic tool for detecting fatigue damages or partial failures in a vibrating structure, or for designing such structures. A very popular example is the control of rotating machinery where rotors are the cause of vibrations in the structures ([24] [25]) ; another important example is the detection of fatigues in an offshore platform.

Most of the techniques used in vibration monitoring try more or less to identify the modal characteristics of the structure through its dynamic responses. Two approaches are available :

-- *using artificial excitation :* periodic excitations with adjustable frequency, or dirac pulses ; such an approach is frequently used for the design of ground and air vehicles [26] but is difficult to use aboard an offshore platform (although it has been successfully applied in some experimental cases).

-- *using natural excitation :* this would be especially convenient in the case of offshore platforms where the seawaves action is such a natural and permanent excitation. In this case identifying the modal characteristics of the structure is a much harder task than in the case of artificial excitation ; a brief review of the previously used techniques is done in appendix in the context of the modelling of offshore platforms.

In this paper, the problem we are interested in is motivated by the detection of fatigues in an offshore platform. The approach is the following. Assume a fatigue (one or several failures) occurs in the structure ; this should result in a modification of some of the modal characteristics of the structure. Thus, if a number of measurements of the dynamic behaviour are available (for example through accelerometers or gauges), and enable the identification of some modal characteristics of the structure, they would provide us with a way of detecting changes in these characteristics, which could be interpreted in terms of the change of the mechanic properties.

In this approach the difficulties are the following :

1) - An offshore platform is a fairly complex vibrating system, and should be considered as *an infinite order linear system with finite* (maybe large : 10 to 100 !) *dimensional observation.* Low order approximations are not sufficient for our purposesince the detection of fatigue is known to require the identification of high order modes ([27]) ; it is hoped that the identification of, say, 20 modes would be convenient for detecting worse failures.

2) - As a natural excitation, the swell effect is difficult to identify. It maybe assumed that, restricted on the bandwith of 1-7 Hz we are interested in, the effect of the swell on the vibrating structure is equivalent to a *nonstationary white noise* (i.e. a white noise with time-varying covariance matrix), since it is mainly originated by secondary wave actions : slamming phenomena, transient vortex shedding,...

The paper is organized as follows. In section II the model of our system is introduced, and the consequences of the previously mentioned difficulties are discussed. In section III the problem of the nonstationary excitation is analysed as a nonstationary stochastic realization problem ; consistency results are stated in this context for the Instrumental Variable method (see [3] for a similar result about the Ho-kalman method). In section IV the problem of finite order approximation of infinite order systems is investigated through the asymptotic theroy of PadéApproximants. The algorithms are given in section V, where a simple geometric approach is given for deriving lattice form algorithms. The section VI is devoted to the experimental results : results are reported on real data coming from offshore platforms, and also on simulated data corresponding to a complex system of masses interconnected by springs.

II - OBTAINING THE MODEL

(II.1) - Continuous time model, and statement of the problem

Denoting by p the derivative operator, and by s the continuous time, we choose the following model for describing our system :

$$(II.1) \quad \begin{cases} (Mp^2 + Cp + K)\, Z_s = E_s \\ y_s = L\, Z_s , \end{cases}$$

where the first equation is the classical second order system encountered in vibration mechanics, and L is the observation matrix : E_s is a *nonstationary* white noise with *time-varying* covariance matrix $Q_E(s)$; the measurement noise is neglected. The corresponding state space model is :

$$(II.2) \quad \begin{cases} pX_s = A\, X_s + B_s \\ y_s = HX_s \end{cases}$$

where

$$(II.3) \quad \begin{cases} X_s = \begin{pmatrix} Z_s \\ pZ_s \end{pmatrix} , \quad A = \begin{pmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{pmatrix}, \quad B_s = \begin{pmatrix} 0 \\ M^{-1}\, E_s \end{pmatrix} \\ H = (L\ ,\ 0). \end{cases}$$

The matrix A is assumed to be *asymptotically stable* and has the following spectral decomposition :

$$(II.4) \quad \begin{cases} A = U\, D\, U^{-1} \\ U = \begin{pmatrix} \Phi & \bar{\Phi} \\ \Phi\Delta & \bar{\Phi}\bar{\Delta} \end{pmatrix} , \quad D = \begin{pmatrix} \Delta & 0 \\ 0 & \bar{\Delta} \end{pmatrix}, \end{cases} \quad (1)$$

---

(1) - *The superscript " $\bar{}$ " means complex conjugate.*

where Φ is the matrixwhose columns are the vibration modes, and Δ contains the *poles* of the system. The problem we are interested in is the following :

PROBLEM 1 : Identify the matrices Δ and $H\,\Phi$

Note that Δ contains the poles of the signal, where-as the matrix $H\,\Phi$ contains the *observed* part of the vibration modes ; indeed, the whole matrix Φ cannot be recovered by observing the signal only, since a change of bases in the strate space modify Φ , while keeping unchanged the matrix $H\,\Phi$.

(II.2) - The discrete time model ; the problems of model reduction and nonstationary excitation

Since we are interested in discrete time identification of the system, let us introduce the discrete time system obtained by sampling *(II.2)* at a sampling rate $1/\delta^o$; and denote by t the corresponding discrete time :

$$(II.5)\quad \begin{cases} X_{t+1} = FX_t + V_{t+1} \;, \quad \mathrm{Cov}\, V_{t+1} = Q_t \\ Y_t = HX_t \end{cases}$$

where H is given in *(II.3)*, and

$$(II.6)\quad \begin{aligned} F &= e^{\delta^o . A} \\ Q_t &= \int_{t.\delta^o}^{(t+1)\delta^o} e^{As}\, Q_B(s)\, e^{A^T s}\, ds \;, \quad Q_B(s) = \left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & M^{-1} Q_E(s) M^{-T} \end{array}\right) \end{aligned}$$

Note that the discrete-time system *(II.5,6)* no more is the output of a time invariant all-pole system driven by nonstationary white noise : indeed, "nonstationary" zeros are introduced by the coupling effect due to the sampling in the formula *(II.6)* which gives Q_t .

Now, the basic problem 1 can be translated in this discrete time context in an obvious way, since Δ has to be replaced by exp $(s^\circ \Delta)$, whereas Φ remains unchanged. *We have thus to solve a nonstationary* (partial since only H and F are searched for) *stochastic realization problem ; moreover, the identification has to be achieved with a single sample of signal,* since the time-varying covariance matrix Q_t has an unpredictable (and certainly not periodic !) behaviour.

Furthermore the model *(II.1)* by no means can be assumed as an exact one, but rather has to be considered as *a reduced order model,* where as the true order should certainly be considered as infinite. This point has to be taken into account in designing the corresponding identification algorithms.

(II.3) - Derivation of the identification methods

The following remarks will lead us to the appropriate identification procedures :

— since the model *(II.5)* has to be considered as a reduced order model, realization algorithms based upon the selection of independent rows or columns in some Hankel matrix should be prohibited, for reasons of numerical ill-conditioning ; as an example, belonging to this class of methods is the famous Ho-Kalman realization algorithm [*4*].

— Since, in our case (as it will be shown later), no reduced order can be considered as "the most relevant one", it is highly preferable to be able to compare the results of the identification obtained on several models of different orders.

— Among the various realization algorithms of Hankel matrices with model reduction, the most powerful ones involve the use of the so-called

"Hankel norm "approximation by the way of the Singular Value Decomposition of the Hankel matrix ([5] [6] [7]). However, on one hand the models we will work with are very large as it will be shown later, and on the other hand we will need the result of the identification for several models of different orders. Hence it will be highly desirable to get efficient algorithms with the property of being *recursive in the order of the model*. For that reason, we left out the realization algorithms based upon the SVD of the Hankel matrix and preferred to use a ladder form version of the Instrumental Variable (*I.V.*) method, which will be now introduced.

Let us assume, for the model *(II.5)*, that *(II.7)* dim $y_t = d$, dim $x_t = p$.

Let be n the smallest index such that there exists matrices $A_1, \ldots, A_n$ such that *(II.8)* $HF^n = \sum_{i=1}^{n} A_i \; HF^{\,n-i}$.

By Cayley-Hamilton theorem, we get $n \leqslant p$; in fact it is generic that $p/d \leqslant n < p/d + 1$, and this later index will be referred to in the simulations as the *generic order* of the system.

The pair $(\hat{H}, \hat{F})$, where

$$(II.9) \quad \hat{H} = (I, 0 \ldots 0) \qquad \hat{F} = \begin{bmatrix} 0 & I & & \\ & \ddots & \ddots & \\ & & 0 & I \\ A_1 & \ldots & & A_n \end{bmatrix}$$

is a (non-minimal) solution to our problem, in the sense that *the pairs* $(\lambda, H\phi_\lambda)$, *where* λ *are the eigen-values of* F *and* ϕ_λ *the corresponding eigenvectors, belong to the set of the pairs* (λ, ψ_λ) *such that*

$$(II.10) \quad (\lambda^n I - \sum_{i=1}^{n} \lambda^{n-i} A_i) \; \psi_\lambda = 0 ,$$

the inclusion being generally strict, unless $\bar{F}$ is a $p\mathrm{x}p$ matrix, i.e. $n = p/d$.

Then, assuming the model *(II.5)* to be exact, we get for every t and $i \geq 0$:

$$(II.11) \quad y_{t+i} = HF^{i}x_t + \sum_{j=1}^{i} HF^{i-j} \; v_{t+j}$$

(where $\sum_{1}^{0} = 0$ by convention). Hence, using *(II.8)* and the fact that v_t is a white noise, we get the following orthogonality conditions : for every t

$$(II.12) \quad y_{t+n} - \sum_{i=1}^{n} A_i \; y_{t+n-i} \perp span \; \{y_t, \; y_{t-1}, \dots \},$$

where

$$(II.13) \quad X \perp y \quad means \quad \mathbb{E}\,(X\, y^{T}) = 0,$$

$\mathbb{E}$ denoting the expectation.

By using the minimum number of orthogonality constraints in *(II.12)*, we get the :

Instrumental Variable (I.V.) method : Solve for $\bar{A}_1 , \dots , \bar{A}_n$ the orthogonality conditions

$$(II.14) \quad y_{t+n} - \sum_{i=1}^{n} \bar{A}_i \; y_{t+n-i} \perp y_t, \dots, y_{t-n+1}$$

Now by taking into account the whole orthogonality conditions, we get the,

Far Past Projection (F.P.P.) method : Solve (in the least squares sense) for $\bar{A}_1 , \dots , \bar{A}_n$ the orthogonality conditions

$$\text{(II.15)} \qquad y_{t+n} - \sum_{i=1}^{n} \hat{A}_i \, y_{t+n-i} \perp y_t \, , \; y_{t-1} \, , \dots$$

We have just explained what are the principles of the methods we shall use. The following problems remain to be investigated :

(i).. The orthogonality conditions *(II.14-15)* involve *exact* covariance, which cannot be estimated since the process y_t is nonstationary, whereas only a single sample of them is available ; this point will be investigated in the next section.

(ii).. What kind of approximation is obtained by satisfying *(II.14-15)* when the model *(II.5)* is not exact ? This will be investigated in section IV.

(iii).. Will the solution of *(II.14-15)* provide us with what we are interested in, namely *a solution of (II.8)* ? We will now give a primary answer to this question by investigating briefly the wellknown *stationary case,* thus assuming for the moment $Q_t = Q$.

The stationary case : a controllability condition

For $n \geq 0$, set

$$\text{(II.16)} \qquad \Gamma_n = \mathbb{E} \, y_{t+n} \, y_t^T \, ,$$

and introduce the Hankel matrices

$$\text{(II.17)} \qquad \mathcal{H}_{n,N} = \begin{bmatrix} \Gamma_0 & \cdots & \Gamma_n & \cdots & \Gamma_N \\ \vdots & & & & \vdots \\ \Gamma_n & \cdots & \Gamma_N & \cdots & \Gamma_{N+n} \end{bmatrix}$$

given matrices H,F,G , denote by

$$(II.18)\qquad \mathcal{O}_n(H,F) = \begin{bmatrix} H \\ HF \\ \vdots \\ HF^n \end{bmatrix}$$

$$\mathcal{C}_n(F,G) = [G, FG, \text{---}, F^nG]$$

respectively the observability matrix of the pair (H,F), and the controllability matrix of the pair (F,G). Denoting by p the solution of the Lyapunov equation

$$(II.19)\qquad P = F\,P\,F^T + Q$$

it is easy to see that the Hankel matrix $\mathcal{H}_{n,N}$ of *(II.17)* exhibits the following factorization

$$(II.20)\quad \mathcal{H}_{n,N} = \mathcal{O}_n(H,F)\ \mathcal{C}_N(F,PH^T).$$

On the other hand, the matrices $\hat{A}_1,\ldots,\hat{A}_n$ defined before are the solution of

$$(II.21)\quad [-\hat{A}_n,\ldots,-\hat{A}_1, I]\ \mathcal{H}_{n,N} = 0$$

where $N = n-1$ for the I.V. method, and $N = +\infty$ for the F.P.P. method (solution in the least squares sense in the later case). Hence, from *(II.20)* it appears that *for (II.21) being equivalent to (II.8), it is necessary and sufficient that the pair* (F, PH^T) *be controllable.* But, provided that (H,F) be observable, this later condition is known to be equivalent to the controllability of the pair

$(F, Q^{1/2})$ ([3] [8] [9]).

Finally, *if the pairs (H,F) and $(F,Q^{1/2})$ are respectively observable and controllable, then the I.V. and F.P.P. methods are equivalent to (II.8) (hence, solve our problem),* at least in the stationary case. And it is already known that only the excited modes can be identified on vibrating structures. The same point will be discussed in the nonstationary case in the following section.

III - SOLVING THE NONSTATIONARY STOCHASTIC REALIZATION PROBLEM

In this section, we assume the model *(II.5)* is exact, thus leaving out the problem of model reduction.

Troughout this section, we assume that *a single sample* $y_0, \ldots, y_S$ *of the nonstationary signal* (y_t) *is available*. We will now indicate what is the implementation of the I.V. and F.P.P. method in this context. For $n \geq 0$, set

$$\text{(III.1)} \qquad \Gamma_n(S) = \sum_{t=0}^{S} y_{t+n} \, y_t^T ,$$

with $y_t = 0$ by convention for $t < 0$ or $t > S$.

Note that *there is no asymptotic behaviour for* $\Gamma_n(S)$ *as* $S \to \infty$ since y_t is a sample of a nonstationary process. Denote by $\mathcal{H}_{n,N}(S)$ the Hankel matrix built as in *(II.17)*, but using the empirical covariance matrices $\Gamma_n(S)$. Then the practical implementation of the I.V. and F.P.P. methods in the nonstationary case is the following :

Instrumental Variable method : solve for $A_1, \ldots, A_n$ the linear system

$$\text{(III.2)} \quad [-A_n, \ldots, -A_1, I] \, \mathcal{H}_{n,n-1}(S) = 0$$

and denote by $\hat{A}_1(S), \ldots, \hat{A}_n(S)$ a solution.

Far Past Projection method of order N : solve for $A_1, \ldots, A_n$ in the least squares sense the linear system

$$(III.3) \quad [-A_n, \ldots, -A_1, I]\ \mathcal{H}_{n,N}\ (S) = 0,$$

and denote by $\hat{A}_1^N\ (S), \ldots, \hat{A}_n^N(S)$ a solution.

The analysis of these methods is done in [3], where the following consistency result is proved :

<u>ASSUMPTION A 1</u> *(i)* <u>The matrix</u> Q_t <u>is bounded uniformly in</u> t

(ii) <u>The pair</u> (H,F) <u>is observable, and there exists a matrix</u> G^*, <u>such that the pair</u> (F,G^*) <u>be controllable, satisfying the following inequality</u>

$$(III.4) \quad Q_t \geq G^* G^{*T}.$$

The following assumption is much weaker than *(III.4)*, since it allows drastic changes in the geometry of the excitation matrix Q_t :

<u>ASSUMPTION A2</u> : <u>Same as A1, except for</u> *(III.4)* <u>which is replaced by the following condition : with probability 1, we have for</u> S <u>large</u>

$$(III.5) \quad \frac{1}{A_S} \sum_{t=0}^{S} V_{t+1} V_{t+1}^T \geq G^* G^{*T},$$

<u>where</u>

$$(III.6) \quad A_S = \sum_{t=0}^{S} \| X_t \|^2$$

<u>is assumed to satisfy</u> $A_S \to +\infty$ <u>w.p.1</u> <u>as</u> $S \to \infty$.

It is proved in [3] that assumptions A1 implie assumptions A2. The following result is also proved :

<u>THEOREM 1</u> : <u>Let the assumptions A1 or A2 be in force. Then there exists an integer</u> N, <u>depending only upon the bound on</u> Q_t <u>and on</u> G^{*}, <u>such that the F.P.P. method of order</u> N <u>is consistent in the sense that</u>

$$\text{(III.7)} \qquad \lim_{S \to \infty} \sum_{i=1}^{n} (\hat{A}_i^N(S) - A_i)\, HF^{n-i} = 0 \quad w.p.\ 1,$$

<u>where</u>

$A_1, \ldots, A_n$ <u>is an arbitrary solution of</u> *(II.8)*.

This is the desired answer to our problem : *the FPP method is robust in presence of nonstationary driving noise*. Furthermore, it is seen in [3] that the accuracy of the metod (i.e. accuracy of the identification of the various modes) is related to the well conditionning of the empirical controllability matrix (see *(II.18)* for the notation)

$$\text{(III.8)} \qquad \mathcal{E}(F, P_S H^T) \ , \quad \text{where} \quad P_S = FP_S F^T + \left(\frac{1}{A_S} \sum_{0}^{S} v_{t+1}\ v_{t+1}^T\right)$$

which turns out to be related to the conditioning of the controllability matrix $\mathcal{E}(F, G_S)$, where

$$\text{(III.9)} \qquad G_S\, G_S^T = \frac{1}{A_S} \sum_{0}^{S} v_{t+1}\ v_{t+1}^T \, .$$

We reobtain the fact that a good identification of a given mode requires a good averaged excitation of this mode during the identification period.

The conclusion of this section is thus the following : for the stochastic realization of a nonstationary process from which a (long) single sample is available, proceed as in the stationary case, but replacing the theoretical covariance function by the single sample estimated one. A similar result is proved in [3] for the classical Ho-Kalman algorithm. Let us emphasize that such a c¯aim would be wrong when using other identification techniques like maximum likelihood estimation or spectral estimation via for example a Burg method : in these cases, zeros are also identified on the signal y_t , which is of no significance in our case since these zeros, due to the effect of the pertubation, are themselves nonstationary. See [3] for a bibliography on the topic of nonstationary stochastic realization.

IV - THE PROBLEM OF MODEL REDUCTION : AN APPLICATION OF THE ASYMPTOTIC THEORY OF PADE APPROXIMANTS

In this section, we will restrict ourselves to the stationary scalar case.

Let us first recall that no stability is guaranteed when using neither the I.V. nor the F.P.P. method with incorrect models ([10]). In this section we are interested in investigating the kind of approximation we get using these methods on infinite order processes.

Padé Approximants [11]

Given a formal power serie

$$(IV.1) \qquad \delta(z) = \sum_{k \geq 0} r_k z^k ,$$

the rational fraction

$$(IV.2) \qquad \delta_{n,p}(z) = \frac{\sum_0^p b(k) z^k}{\sum_0^n a(k) z^k} \triangleq \frac{B_{n,p}(z)}{A_{n,p}(z)}$$

is an (n,p)- Padé Approximant of δ if

$$(IV.3) \qquad A_{n,p}(z)\, \delta(z) - B_{n,p}(z) = \sum_{k > p+n} \tilde{r}_k z^k .$$

The polynomials $A_{n,p}$ and $B_{n,p}$ are defined by the following Toeplitz system, where, *by convention*, $r_n = 0$ *for* $n<0$:

$$(IV.4) \qquad \begin{bmatrix} r_0 & & & \\ r_1 & r_0 & \ddots & \\ \vdots & & & \\ r_p & \cdots & \cdots & r_{p-n} \\ \hline r_{p+1} & \cdots & \cdots & r_{p+1-n} \\ \vdots & & & \vdots \\ r_{p+n} & \cdots & \cdots & r_p \end{bmatrix} \begin{bmatrix} a^0_{n,p} \\ \vdots \\ a^n_{n,p} \end{bmatrix} = \begin{bmatrix} b^0_{n,p} \\ \vdots \\ b^p_{n,p} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Now let us consider a stationary scalar signal (y_t), and denote by $\Gamma_n = \mathbb{E}\,(y_{t+n}\ y_t)$ its covariance function. Denote by

$$(IV.5) \qquad f(z) = \frac{1}{2}\Gamma_0 + \sum_{k>0} \Gamma_k\, z^k$$

its half-spectrum, or impedance function. Then, *for* $p \geq n-1$, it is clear from (IV.4) that *the denominator* $A_{n,p}$ *of the* (n,p) *- Padé Approximant of* f *is also the solution of the equivalent Hankel system*

$$(IV.6) \quad [a(n),\ldots,a(o)] \begin{bmatrix} \Gamma_{p-n+1} & \cdots & \Gamma_{p} \\ \vdots & & \Gamma_{p+1} \\ \Gamma_{p} & & \vdots \\ \Gamma_{p+1} & \cdots & \Gamma_{p+n} \end{bmatrix} = 0$$

which corresponds to a delayed I.V. method (*P = n-1* for the method of section II) *for the stochastic realization of* (Y_t). As a consequence no meaningful metric can be associated to be I.V. method, since the Padé Approximants are known to be local (compare with the approximation by all-pole models, where a natural metric is available [12] [13], which corresponds to $p = 0$ in the system *(IV.6)*.

However the asymptotic theory of Padé Approximants gives some insight on the kind of approximation one gets using the (delayed) I.V. method. Denote by α_1 , $\alpha_2, \ldots$ the poles of the signal (Y_t), ordered by the modules :

$$1 < |\alpha_1| \leq |\alpha_2| \leq \ldots$$

We have the following theorem :

<u>THEOREM 2</u> (Montessus de Ballore, 1902, [10] [11] [14]) : *for n fixed, the roots of* $A_{n,p}(z)$ *converge to the set* $\{\alpha_1,\ldots,\alpha_n\}$ *as* $p \to +\infty$.

Hence the I.V. method asymptotically ($p \to +\infty$) selects the most oscillatory poles of the signal, which is an interesting property. No similar result is available for the F.P.P. method ; nevertheless theorem 2 gives also information on the later method, since the F.P.P. method corresponds to some compromise between the solution of different I.V. methods corresponding to different delays p . As it has been pointed out to us by M. Morf ([2]), the FPP method is nothing but a covariance method applied to the sequence of covariance matrices $(\Gamma_n(S))_{n>p}$, which is asymptotically equivalent to a Levinson method when the number of constraints grows ; hence, for N large, the FPP method of order N ensures approximately the stability of the identified system. This is confirmed by the simulations.

Unfortunately, as it will be shown in the experiments, the theorem of Montessus de Ballore is not robust when the exact covariances are replaced by estimates of them.

V - A SIMPLE DERIVATION OF LADDER FORM ALGORITHMS

Since we are interested in identifying models of different orders, it is desirable to derive algorithms which are recursive with respect to this order.

(V.1) - Ladder form algorithms for the Instrumental Variable method

We shall derive these algorithms in a stationary context using a very simple geometrical framework. For further information on exact algorithms for the "non stationary" case (i.e. taking into account the boundary effects arising when finite samples are involved), see [15] [16] [17] [18].

Consider a pair (y_t , u_t) of jointly stationary processes ; the process u_t will be referred to as the *Instrumental Variable process.*

([2]) - *Private communication*

Introduce the following *forward* and *backward residuals*, where the matrix coefficients are adjusted so that the following orthogonality conditions be satisfied (see *(II.13)* for the notation $\perp$) :

$$
(V.1)\quad
\begin{aligned}
e_t(n,p) &\triangleq y_t - \sum_{i=1}^{n} A^i_{n,p}\, y_{t-i} \perp u_{t-p-1},\dots,u_{t-p-n} \\
b_t(n,p) &\triangleq y_{t-n} - \sum_{i=1}^{n} B^i_{n,p}\, y_{t-n+i} \perp u_{t-p},\dots, u_{t-p-n+1}
\end{aligned}
$$

with, by convention, $e_t(o,o) = b_t(o,o) = y_t$.

Note that the problem *(II.14)* we are interested to solve is embedded in the family of problems *(V.1)* , corresponding to the case $u_t = y_t$, $p = n-1$.

Denote by

$$
(V.2)\quad \mathcal{Y}^t_s \triangleq \text{Span}\,\{y_s,\dots, y_t\}
$$

the linear space spaned by the variables $y_s,\dots, y_t$ for $-\infty \leq s \leq t \leq +\infty$ Then *the following conditions characterize the residuals up to a constant normalization matrix :*

$$
(V.3)\quad
\begin{aligned}
&e_t(n,p) \in \mathcal{Y}^t_{t-n} \text{ and } e_t(n,p) \perp \mathcal{U}^{t-p-1}_{t-p-n} \\
&b_t(n,p) \in \mathcal{Y}^t_{t-n} \text{ and } b_t(n,p) \perp \mathcal{U}^{t-p}_{t-p-n+1} .
\end{aligned}
$$

Three different recursions may be obtained :

-- p *fixed*, $n \to n+1$

-- n *fixed*, $p \to p+1$

-- $n \to n+1$, $p \to p+1$.

We will derive in details the first recursion, and give only the results for the other ones.

From *(V.1)* and the orthogonality conditions in *(V.3)* we get

$$(V.4)\qquad \begin{aligned} &e_t(n+1,p) - e_t(n,p) \in \mathcal{Y}_{t-n-1}^{t-1} \\ &e_t(n+1,p) - e_t(n,p) \perp \mathcal{U}_{t-p-n}^{t-p-1} \end{aligned}$$

so that, by the characterization *(V.3)*, there exists a matrix $K_{n,p}$ such that

$$(V.5)\qquad e_t(n+1,p) - e_t(n,p) = - K_{n,p} \quad \delta_{t-1}(n,p) \;;$$

the matrix $K_{n,p}$ is entirely defined by the orthogonality condition which was missed in *(V.4)*, namely

$$e_t(n+1,p) \perp u_{t-p-n-1} \,,$$

which implies

$$(V.6)\qquad K_{n,p} = \langle e_t(n,p),\, u_{t-p-n-1}\rangle \langle \delta_{t-1}(n,p),\, u_{t-p-n-1}\rangle^{-1}$$

where, by definition

(V.7) $\langle X, Y\rangle$ de notes the matrix $\mathbb{E}\, X\, Y^T$.

The formulas *(V.5)* and *(V.6)* give together the first formula of the ladder form recursion. The other one is obtained by applying the same method to $\delta_t(n+1,p) - \delta_{t-1}(n,p)$.

As customary, these recursions can be expressed using the polynomials

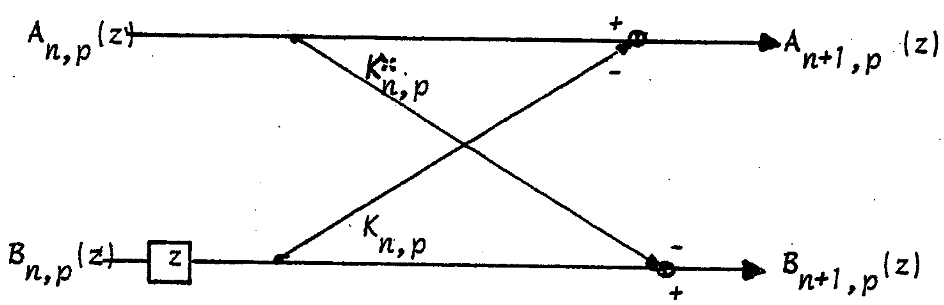
$$(V.8)\qquad \begin{aligned} &A_{n,p}(z) \triangleq I - \sum_{i=1}^{n} A_{n,p}^{i} \; z^{i} \;,\; A_{0,0} = I \\ &B_{n,p}(z) \triangleq z^{n} I - \sum_{i=1}^{n} B_{n,p}^{i} \; z^{n-i} \;,\; B_{0,0} = I, \end{aligned}$$

whereas the matrix reflection coefficients $K_{n,p}$ can be expressed using the covariance matrices

$$\text{(V.9)} \qquad \Gamma_k \triangleq \langle y_t \, , \, u_{t-k} \rangle = \mathbb{E} \; y_t \; u_{t-k}^T \, .$$

The three resulting recursions are the following [10]-[1] :

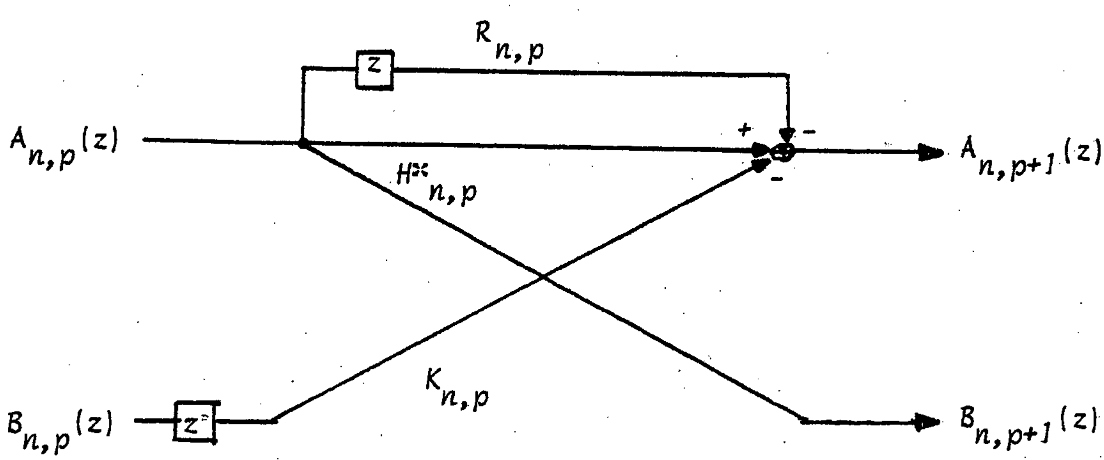
-- $\underline{p \text{ constant, } n \rightarrow n+1}$



where the two different matrix reflection coefficients are

$$\text{(V.10)} \qquad \begin{aligned} K_{n,p} &= \Big(\Gamma_{p+n+1} - \sum_{i=1}^{n} A^{i}_{n,p} \, \Gamma_{p+n+1-i}\Big) \Big(\Gamma_p - \sum_{i=1}^{n} B^{i}_{n,p} \, \Gamma_{p+i}\Big)^{-1} \\ K^{*}_{n,p} &= \Big(\Gamma_{p-n-1} - \sum_{i=1}^{n} B^{i}_{n,p} \, \Gamma_{p-n-1+i}\Big) \Big(\Gamma_p - \sum_{i=1}^{n} A^{i}_{n,p} \, \Gamma_{p-i}\Big)^{-1} \end{aligned}$$

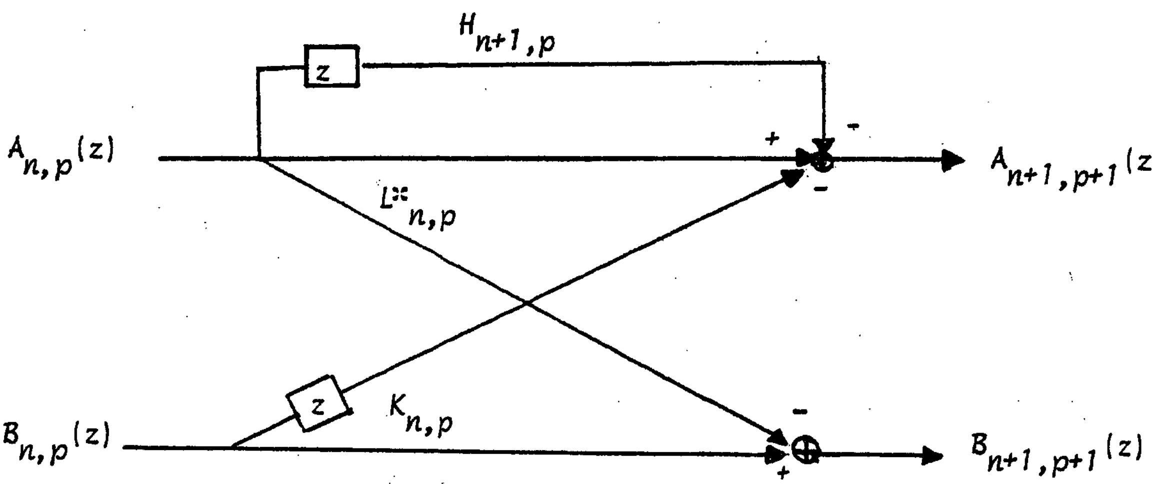
-- $\underline{n \text{ constant, } p \rightarrow p+1}$



where $K_{n,p}$ is defined in *(V.10)*, where-as

$$(V.11) \quad H^{*}_{n,p} = -(A^{n}_{n,p})^{-1}, \quad R_{n,p} = K_{n,p} H^{*}_{n,p}.$$

-- <u>$n \to n+1,\ p \to p+1$</u>



where $K_{n,p}$ and $K^{*}_{n,p}$ are defined in *(V.10)*, whereas

$$L^{*}_{n,p} = (K^{*}_{n,p})^{-1}$$

$$(V.12) \quad H_{n+1,p} = \tilde{H} \cdot \left(\Gamma_{p+n+1} - \sum_{i=1}^{n} A^{i}_{n,p}\, \Gamma_{p+n+1-i}\right)^{-1}$$

$$\tilde{H} = \left(\Gamma_{p+n+2} - \sum_{i=1}^{n} A^{i}_{n,p}\, \Gamma_{p+n+2-i}\right) - K_{n,p}\left(\Gamma_{p+1} - \sum_{i=1}^{n} B^{i}_{n,p}\, \Gamma_{p+1+i}\right).$$

The first recursion is nothing but the Trench algorithm for the block-Toeplitz case ([1] [10] [19]), where-as the other recursions seem to be new.

When applied with $U_t = y_t$, these recursions provide us with algorithms for getting the pole part $A_{n,p}(z)$ of an approximate ARMA model of orders (n,p) for the process y_t. Although our model *(II.5)* corresponds to an ARMA $(n,n-1)$ model (compare *(II.14)* with *(V.1)*), we will try to adjust on the signal y_t several models ARMA (n,p) with different n and p's, but with the constraint $p \geq n-1$ in view of *(II.12)*.

(V.2) - Recursions for the Far Past Projection method

We will give recursions over the number N of constraints which is taken into account in the FPP method of order N *(cf (II.15), (II.21)* and *(III.3))*, whereas the "order" n of the model is kept constant. We shall thus obtain recursions of different kind as for the I.V. method : these are not ladder form recursions. Starting from *(V.1)*, we add further orthogonality constraints, for obtaining the corresponding FPP (N) method : the A'_i s are the solution in the least squares sense of the orthogonality constraints

$$(V.13) \quad y_t - \sum_{i=1}^{n} A_i \, y_{t-i} \perp u_{t-p-1}, \ldots, u_{t-p-N}$$

where $N > n$. The corresponding Toeplitz system is

$$(V.14) \quad A(N) . \mathcal{T}(N) = L(N),$$

where

$$A(N) = [A_1, \ldots, A_n],$$

$$(V.15) \quad \mathcal{T}(N) = \begin{bmatrix} \Gamma_p & \cdots & \Gamma_{p+N-n} & \cdots & \Gamma_{p+N-1} \\ \vdots & \ddots & & \ddots & \vdots \\ \Gamma_{p-n+1} & & \Gamma_p & \cdots & \Gamma_{p+N-n} \end{bmatrix}$$

$$L(N) = [\Gamma_{p+1} \cdots \Gamma_{p+N}] .$$

The solution of *(V.14)* is given by

$$(V.16) \qquad A(N) = L(N)\ \mathcal{C}(N)\ \left(\mathcal{C}^T(N)\ \mathcal{C}(N)\right)^{-1}$$

and a square root algorithm using orthogonal transformations ([20]) is used for uptdating the square root of the last matrix in *(V,16)* ; see [1] for further details. The initial value of this square root, namely $\mathcal{C}(n)^{-1}$ is obtained through the I.V. method of orders (n,p), using a Gohberg-Semmencul formula ([1] [21]). Fast algorithms are also available for this problem.

This recursion over the number N of constraints can also be combined together with recursions over n and p, by using inversion formulas for partitioned matrices.

(V.3) - Practical implementation

The algorithms *(V.10-11-12)* and *(V.14-15)* are used with $U_t = Y_t$. When a single sample $Y_0, \ldots, Y_S$ of the nonstationary signal Y_t is available, the "true" covariance matrices Γ_k are simply replaced by the empirical covariance matrices $\Gamma_k(S)$ defined in *(III.1)* ; this point was justified in section III. The so obtained results should be considered as valid for sufficiently large S.

Then, the modal characteristics are identified by checking the eigenvalues and eigenvectors of the companion matrix $\bar{F}(S)$ which is built on the estimated matrix coefficients $\bar{A}_i(S)$, as indicated in *(II.9)*. For this purpose, a reduction of $\bar{F}(S)$ to the Hessenberg form is first used, then a Q-R factorization algorithm is repeatidly used until convergence to the desired diagonal form. This part of the algorithm is much time consuming. Some alternatives would be :

1— (i). reduction of the polynomial matrix $I - \Sigma A_i z^i$ to the Hermite form, *(ii)*. extraction of the poles by root finding iterative algorithms,

(iii). computation of the corresponding modes (i.e. vectors of the nullspace). But the classical techniques for the reduction to the Hermite form are not very efficient [22].

2— *(i)*. orthogonalization of the components $(y_t^1, \ldots, y_t^d)$ of the observation y_t in the following sense : set

$$\tilde{y}_t^1 \triangleq y_t^1 - \mathbf{E}\,(y_t^1 \,/\, y_s^i \; ; \; i = 2, \ldots, d \; ; \; s \leq t)$$

(V.16)

$$\tilde{y}_t^k \triangleq y_t^k - \mathbf{E}\,(y_t^k \,/\, y_s^i \; ; \; i = k+1, \ldots, d \; ; \; s \leq t)$$

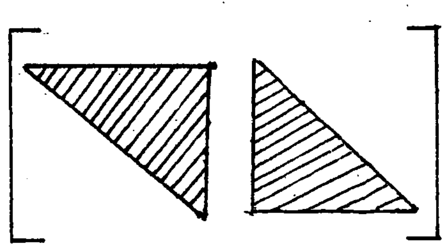
$$\tilde{y}_t^d = y_t^d .$$

This may be achieved efficiently using a Levinson algorithm.

(ii). Apply the I.V. or F.P.P. method to the so-obtained $\tilde{y}_t$ process : the resulting polynomial matrix is in the Hermite form.

(iii). Proceed as suggested in the preceding alternative. Note that the two steps *(i)* and *(ii)* is nothing than a suitable way of performing the reduction of the polynomial matrix $I - \Sigma A_i z^i$ in the Hermite form, since the following factorization is obtained

(V.17) $I - \Sigma A_i z^i$ =



$= U(z) \; L(z)$

where the upper triangular matrix $U(z)$ is unimodular, and corresponds to the step *(i)*, whereas $L(z)$ is the desired Hermite form, which is obtained by the step *(ii)*.

These alternatives were suggested to us by M. Morf ; the second one may be interesting, although we have not use it at the moment.

Nevertheless, it seems that the search of the eigenvalues via the Q-R factorization algorithm may be dramatically improved if good approximations are available for these eigenvalues : but this is indeed the case in our situation, since the identification is performed for various orders and numbers of constraints. Finally such an improvement of the method we have used could be the best solution in our case ; further work is in progress in this direction.

VI - EXPERIMENTAL RESULTS

We shall give results on real as well as simulated data.

(VI.1) - A simulated vibrating system

The *figure 1* shows a simulated system of masses interconnected by springs, where we are interested in modelling the motion of the system along the vertical axis. There are 30 masses, among them 4 are observed. The corresponding system is of the form *(II.5)* with dim $X_t = 60$, and is generically observable, and controllable by any non zero (stationary) excitation. Since proportional damping were synthetized ($C = \alpha K + \beta M$ in *(II.1)*), the modes are real.

Figure 1

(VI.2) - A real vibrating system

The *figure 2* shows a jacket (fixed offshore platform), where the two components N-S and W-E of the acceleramoters C and D were used, thus giving a 4 - dimensional observation.

Figure 2

The *figure 3* shows two pieces of 320 samples located at different places of the same 5800 samples - length record. The signal is clearely non-stationary ; moreover it is (unfortunately !) too coarsely quantized.

Figure 3

(VI.3) - Notations

The experimental results involve

— the presentation of the identified frequencies, together with information on their damping, along the different steps of the procedures (various orders (n,p) for the I.V. method, and different number of constraints for the F.P.P. method).

— the presentation of the identified modes.

Notations for the presentation of the identified frequencies

x axis : frequency (from 0 to π)

y axis : (n,p) = order of the model for I.V. method

(n,p) = (number of matrices A_i, i.e. AR order, MA order + number of further constraints)for the FPP method

| : exact frequencies (for the simulated system) plotted at the bottom

x : unstable poles with $1 \leq |z| < 1.1$

+ : stable poles, damping between 0% and 4%

ı : stable poles, damping between 4% " 6%

. : other stable poles

Presentation of the modes

Given a modal vector $\phi^T = (\phi_1, \ldots, \phi_4)$, where the ϕ_i's are complex numbers, the coordinates of the corresponding normalized modal vector $\hat{\phi}^T = (\phi_1/\phi_2\ ,\ 1,\ \phi_3/\phi_2\ ,\ \phi_4/\phi_2)$ (where, for examples $|\phi_2| \doteq \max |\phi_i|$) are shown in the complex plane as indicated in the *figure 4*, where ϕ_i/ϕ_{max} is plotted at place i.

Figure 4

(VI.4) - The results on the simulated system

The I.V. method

— *Figure 5* : Illustration of the theorem of Montessus de Ballore

- all masses are excited (Q diagonal, with non zero diagonal entrées)
- length of the sample = 5800
- results of an ARMA (15,P) modelling with $P = 15,\ldots,60$

Although a long sample was used, the convergence of the poles is not really satisfactory as P grows : the theorem of Montessus de Ballore is not robust.

— *Figure 6.i* : behaviour of the identified modes

- all masses are excited
- length of the sample = 5800
- results of ARMA (N,N) modelling, with $N = 1,\dots,39$

The *figure 6.i* shows the identified frequencies. Note the band B where two exact frequencies exist (with a gap of $5/1000\pi$) and are indeed recognized with different modes ; note also the band C where a true frequency is recognized after the "exact order" 15 (=60/4), thus suggesting that this highdimensional system behaves like an infinite dimensional one.

The *figures 6.ii, iii, iv* show respectively the identified modes in the frequency bands A, C, D. Note that these modes are consistent in the bands A and C (which correspond to true frequencies), whereas there appear at random in the band D.

The F.P.P. method

— *Figure 7.i and ii* : Comparison between F.P.P. and I.V.

- 3 masses excited
- length of the sample = 5000

The *figure 7.i* shows the result of the F.P.P. method for an AR order of 10 (giving 20 poles) and constraints corresponding to $N = 10$ to 42 in the formula *(III.3)*

The *figure 7.ii* shows the result of the I.V. method for an ARMA (n,n) model with $n = 1$ *to* 22 (giving at most 44 poles).

Note that for $N \sim 40$ all the identified poles are stable in the case of the F.P.P. method, whereas it is not the case for the I.V. method since only 25 stable poles are identified. Recall that identifying meaningless poles is highly undesirable, since the diagonalization is much time consuming.

Figure 8.i, ii, iii : Robustness with respect to a non stationary excitation

. F.P.P. method as in the *figure 7.i*

— *figure 8.i* : . stationary excitation, 3 masses excited, $Q = (Q_{ij})$ with $Q_{ii} = q$, $i = 4,5,6$, $= 0$ elsewhere.
. length of the sample = 600

— *figure 8.ii* : . stationary excitation, all masses excited $Q' = (Q'_{ij})$ with $Q'_{ii} = Q_{ii} + \frac{1}{2} q$
. length of the sample = 600

— *figure 8.iii* : . nonstationary excitation : $Q_t = Q$ for $0 \leq t < 300$, $Q_t = Q'$ for $300 \leq t < 600$, and the transient period after 300 was kept.
. length of the sample = 600

It appears in *8.iii* a compromise between the results of the two stationary cases, without further degradation due to the nonstationarity of the excitation, which is a very satisfactory result. Note that *model reduction* has been furthermore enforced.

(VI.5) - The results on the real system

— *figure 9* : . length of the sample = 5800, among them two different parts of 300 samples are shown in the *figure 3*; note the non stationary character of the signal.

. the figure shows the result of the FPP (n,N) method *(cf (III.3))* with $n = 10$ and $N = 10,\ldots,15$; $31,\ldots,35$; $51,\ldots,55$; $71,\ldots,75$.

For further information, see [1].

CONCLUSION

We have considered the problem of the identification of the modal characteristics of a vibrating system, subject to a noncontrolled and unknown excitation, as a stochastic realization problem with nonstationary excitation. The robustness of two already known methods (I.V and F.P.P methods) has been investigated with respect to the nonstationary character of the excitation, as well as the necessary model reduction. Efficient algorithms were developped for this purpose.

The main conclusions are the following :

1)— This method seems to provide a substantial improvement over the already used ones, which are based upon the analysis of the spectral density matrix of the signal (see [1] and the appendix) : very close poles coresponding to different modes can be recognized, and separated ; furthermore, the robustness with respect to nonstationary excitation has been established, which was not the case for the methods dealing with the spectral density matrix. As a consequence, it appears that an identification based on a long nonstationary sample could be preferable compared to a collection of identification based on small quasistationary samples, since the result of the identification could be less sensitive with respect to the variations of the excitation.

2)— The extraction of roots and eigenvectors in a high degree/high dimensional polynomial matrix leads to time consuming computations. Fast and stable algorithms should be developped for this purpose ; some promising ways are indicated in this paper.

AKNOWLEDGMENT : The authors are indebted to Martin Morf for very fruitful discussions.

APPENDIX : A comparison with methods based upon the diagenalization of the spectral density matrix (S.D.M. method)

This method has been frequently used for the purpose of obtaining the modes of a vibrating system subject to a natural excitation [10] [23].

Description of the S.D.M. method

— *STEP 1 :* Estimation of the spectral density matrix $S(e^{i\Theta})$ of the signal (y_t).

Various techniques are used for this purpose :
Fourier transforms, Burg method, Levinson method, for example.

— *STEP 2 :* Check the local maxima of the function $\Theta \rightarrow T_r\ (S(e^{i\Theta}))$; denote by Θ_0 one of these maxima (other measures could be used for detecting "large" symmetric matrices).

— *STEP 3 :* Let $\phi(\Theta_0)$ be the eigenvector corresponding to the greatest eigenvalue of $S(e^{i\Theta_0})$. Then the pairs $(\Theta_0,\ \phi\ (\Theta_0))$ are the identified frequencies and corresponding modes.

Connection with the method proposed in this paper, under the assumption that $Q_t = Q$ be constant.

Select a pair (λ_0, ϕ_0) (see (II.10)), where λ_0 is an eigenvalue of F, and ϕ_0 the corresponding eigenvector. Let us introduce the following assumption :

ASSUMPTION A3 : The pole λ_0 lies near to the unitcircle, and is sufficiently isolated from other eigenvalues of F.

As a consequence, $\bar{\lambda}_0^{-1}$ is also near to the unit circle, and $|\lambda_0 - \bar{\lambda}_0^{-1}|$ is small. Set

$$(A.1) \qquad \delta_z = |1 - z\lambda_0^{-1}|$$

there is $|z| = 1$ such that δ_z be small, thanks to the assumption. Denote by

$$(A.2) \qquad F = U D U^{-1}$$

the spectral decomposition of F, and assume λ_0 is the first entry of the diagonal matrix D, so that ϕ_0 is the first column of U. Then, straightforward manipulations leads to the following result : for $|z| = 1$ such that $\delta z = |1 - z\lambda_0^{-1}| \ll |1 - z\lambda^{-1}|$, where $\lambda \neq \lambda_0$ denotes an arbitrary eigenvalue of F (such z do exist, thanks to the assumption), we have

$$(A.3) \quad S(z) = \frac{1}{|\delta z|^2} \quad H \quad \left[\phi_0 \mid 0 \ldots 0\right] \quad U^{-1} Q U^{-*} \begin{bmatrix} \phi_0^* \\ 0 \\ \vdots \\ 0 \end{bmatrix} H^T - O\left(\frac{1}{|\delta z|}\right) \quad (^3)$$

where $O(x) \sim x$ as $x \to \infty$, and

$$(A.4) \quad S(z) \triangleq H(I - zF)^{-1} Q (I - z^{-1} F^T)^{-1} H^T$$

is the spectrum of the signal (y_t) (recall the stationarity assumption is in force). Finally, partitioning $U^{-1} Q U^{-*}$ as

$$(A.5) \qquad U^{-1} Q U^{-*} = \begin{pmatrix} q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}$$

---

(3) *"*" denotes hermitian transpose, "-*" denotes inverse of hermitian transpose.*

with q_{11} scalar, and assuming that q_{11} is nonneglectible with respect to the other entries of $U^{-1} Q U^{-*}$ (i.e. *the mode ϕ_0 is sufficiently exited)*, we get for such a z :

$$(A.6) \qquad S(z) = \frac{q_{11}}{|\delta z|^2}(H\phi_0)(H\phi_0)^* + O\left(\frac{1}{|\delta z|}\right).$$

But, in this case, it appears that *(A.6)* is nothing but the degenerate form of the diagonalization of $S(z)$ at $z_0 = e^{i\theta_0}$, which is a peak of $\underset{\sim}{Tr}(S(e^{i\theta}))$ near to λ_0.

Finally, assuming the assumption *(A.3)* in force, and assuming the excitation to be stationary, we get that the S.D.M. method is equivalent to the identification of the modal characteristic $(\lambda_0, H\phi_0)$ via the method we have proposed.

However, in the case two poles are very close, with different corresponding modes, the same argument shows that the diagonalization of $S(e^{i\theta_0})$ $(e^{i\theta_0}$ being near to both considered poles) will degenerate into a decomposition of rank 2 (2 main eigenvalues, the other possibly neglectible), where only the space spaned by the two modes will be recognized with an arbitrary basis. This is highly undesirable, since the so identified modes will appear as numerically unstable.

Furthermore, it is clear that a method based upon the estimated spectral density of (Y_t) is not robust in the case of a nonstationary excitation creating "time varying zeros" in the true spectrum.

See [23] for other already used methods, which seem to be less efficient than the S.D.M. method.

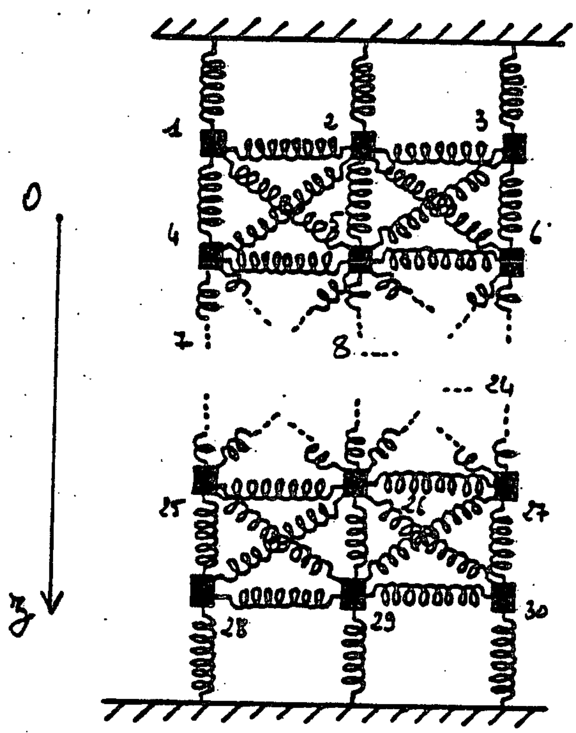


Figure 1 : 30 Masses system used for Synthetic Signals Generation

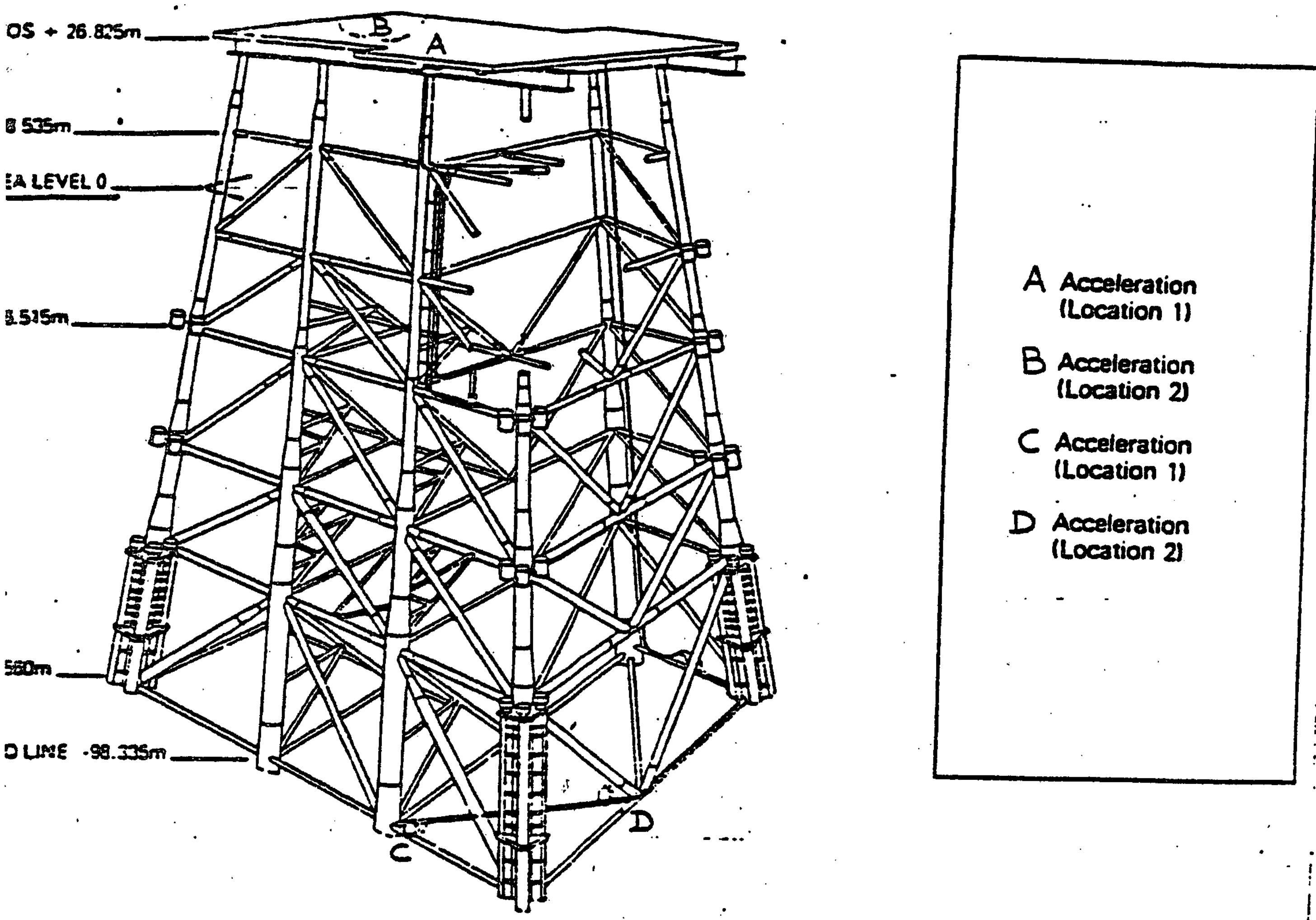


Figure 2 : Platform instrumentation layout

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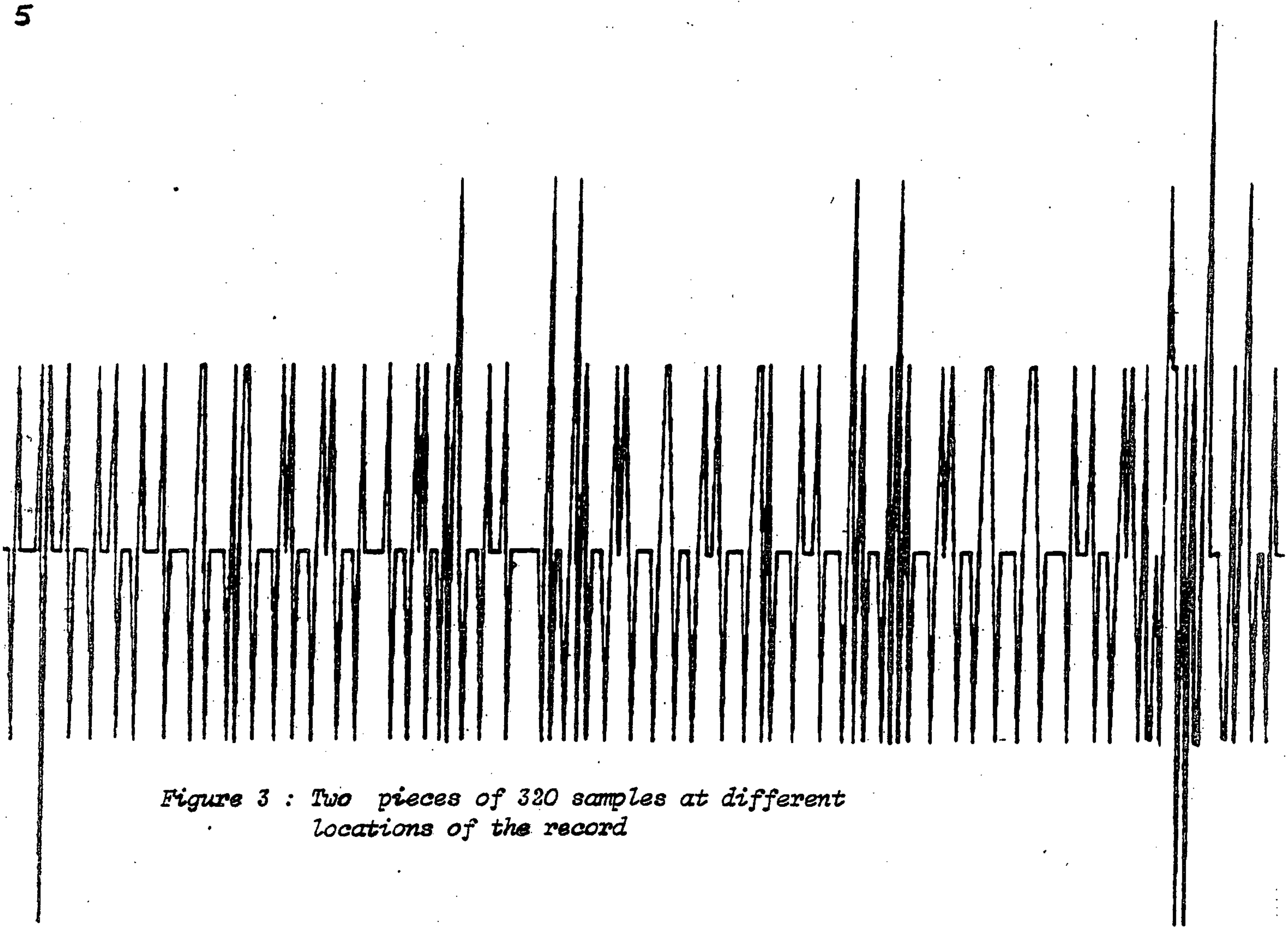

Figure 3 : Two pieces of 320 samples at different locations of the record

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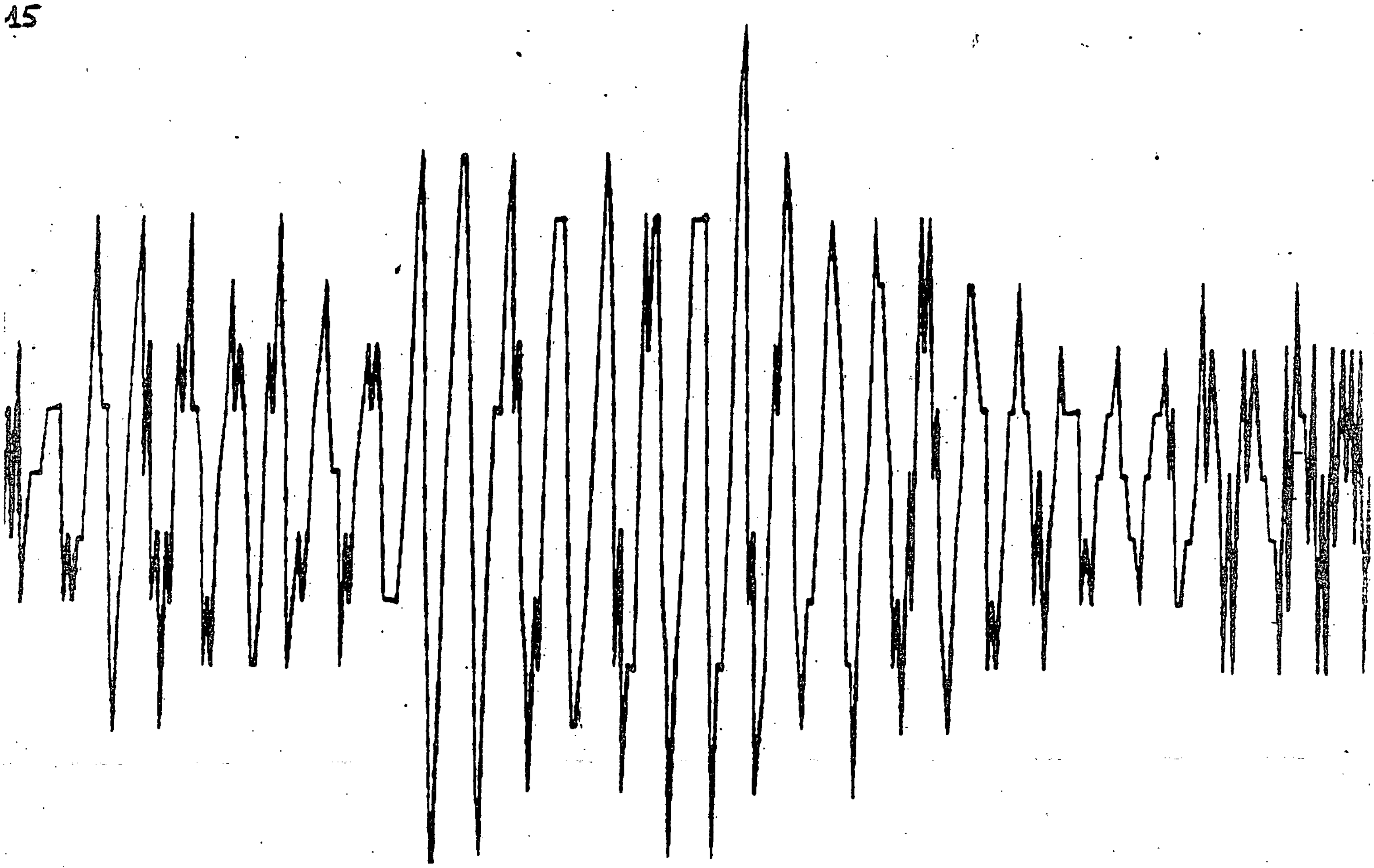

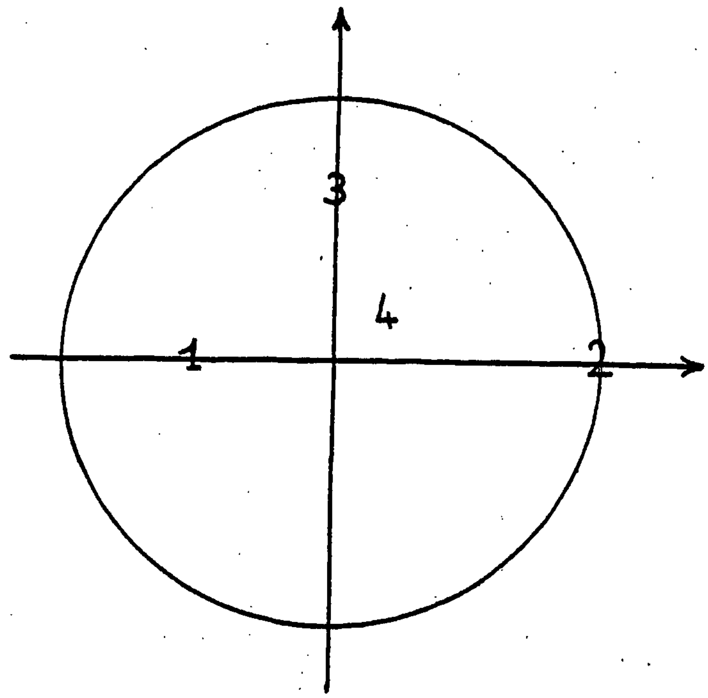

Figure 4 : Presentation of the modes

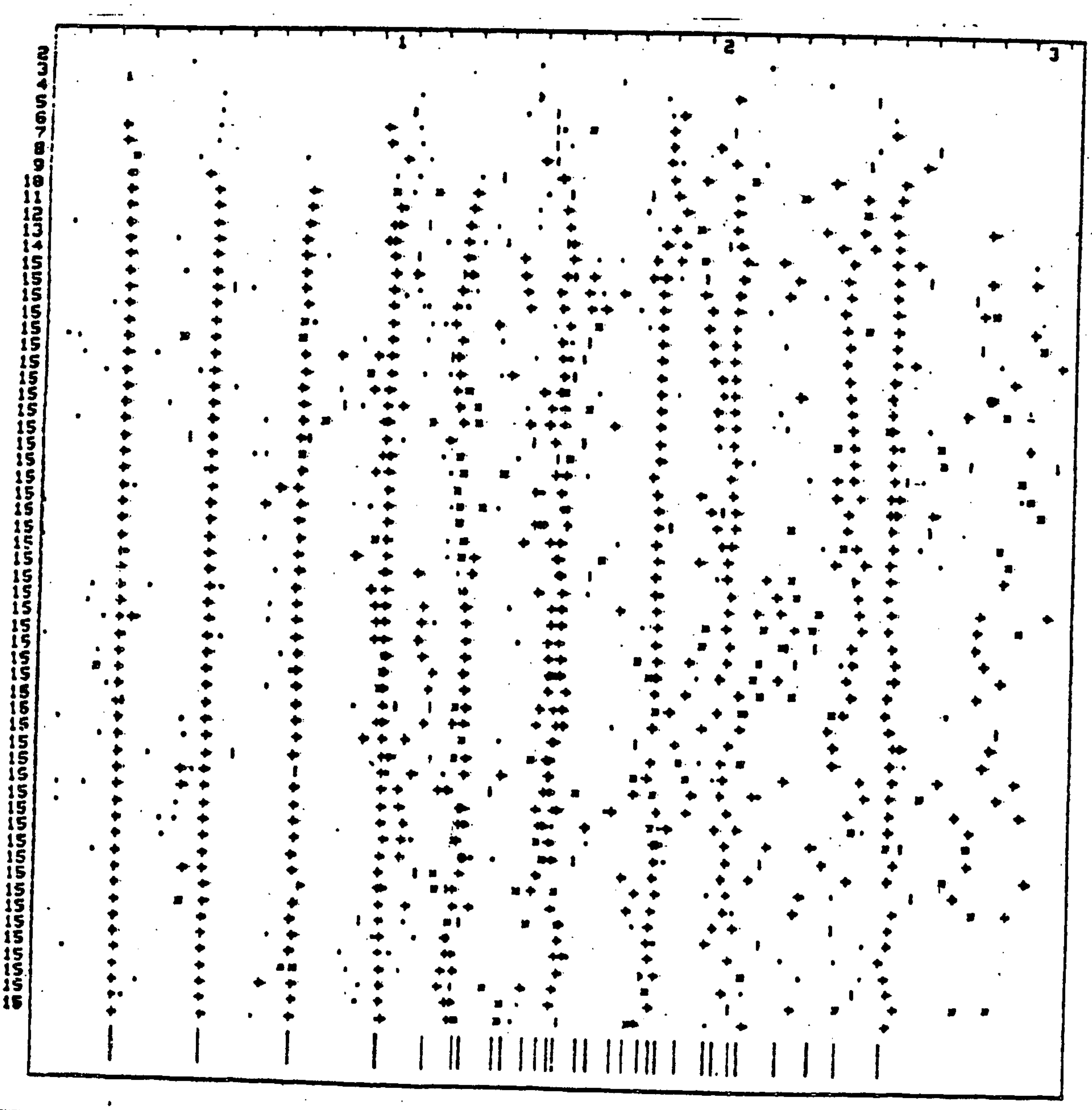

Figure 5 : ARMA (15,P) All masses excited, 5800 samples

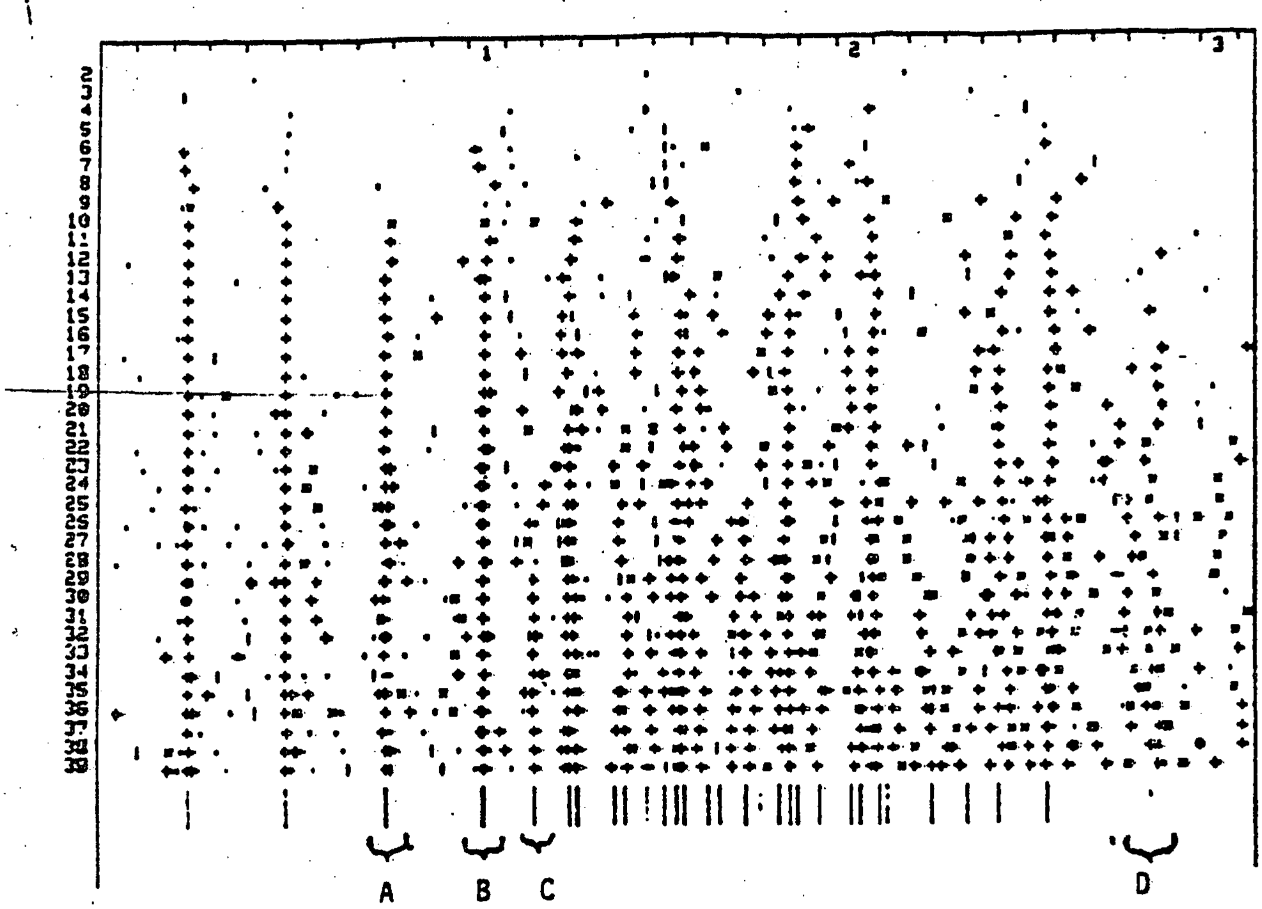

Figure 6.i : ARMA (N,N) All masses excited, 5800 samples

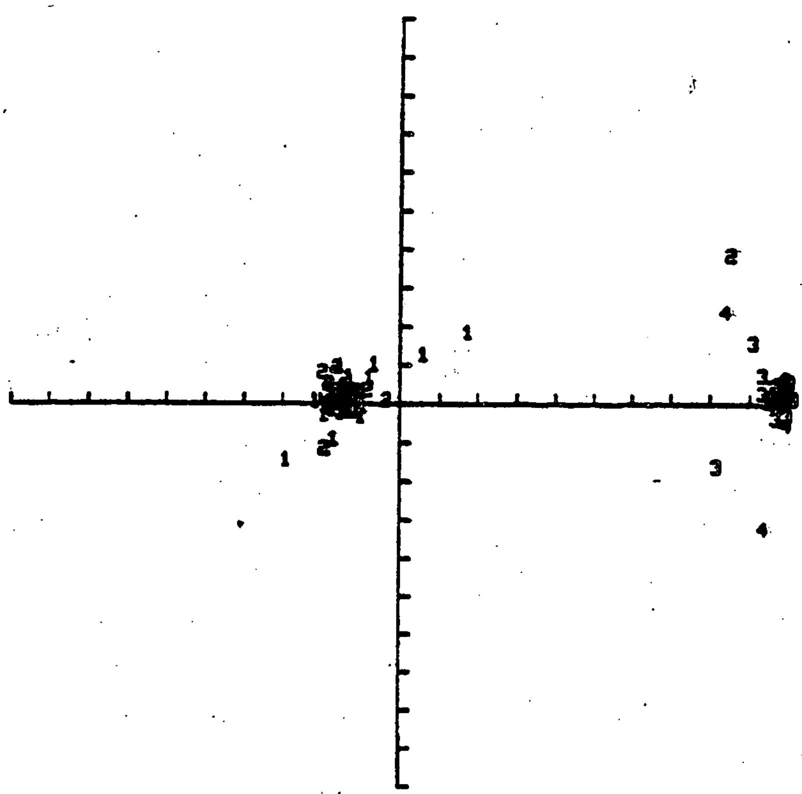

Figure 6.ii : Band A

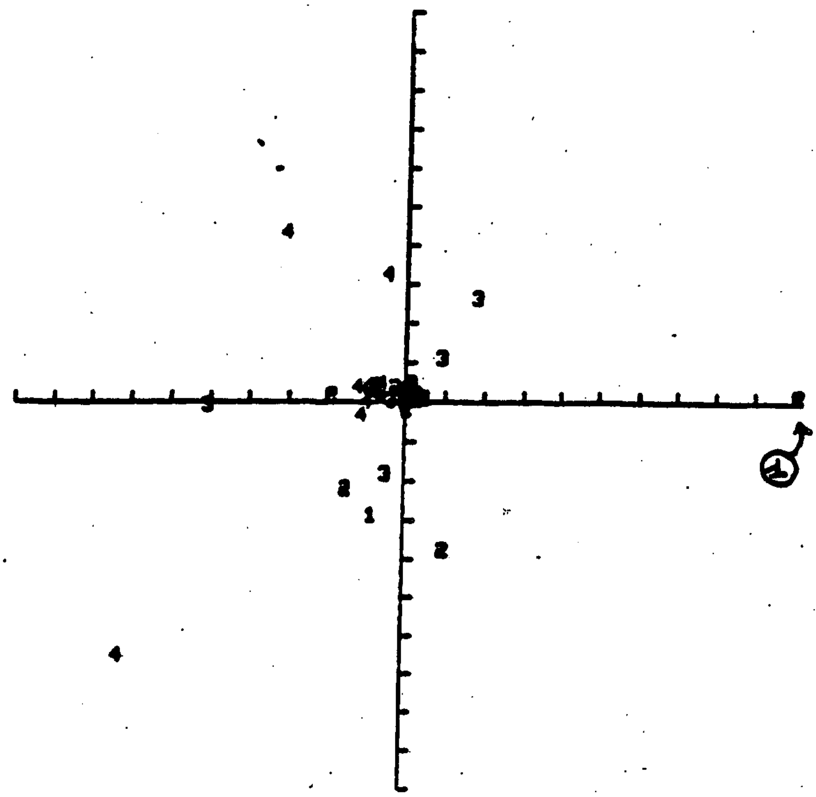

Figure 6.iii : Band C

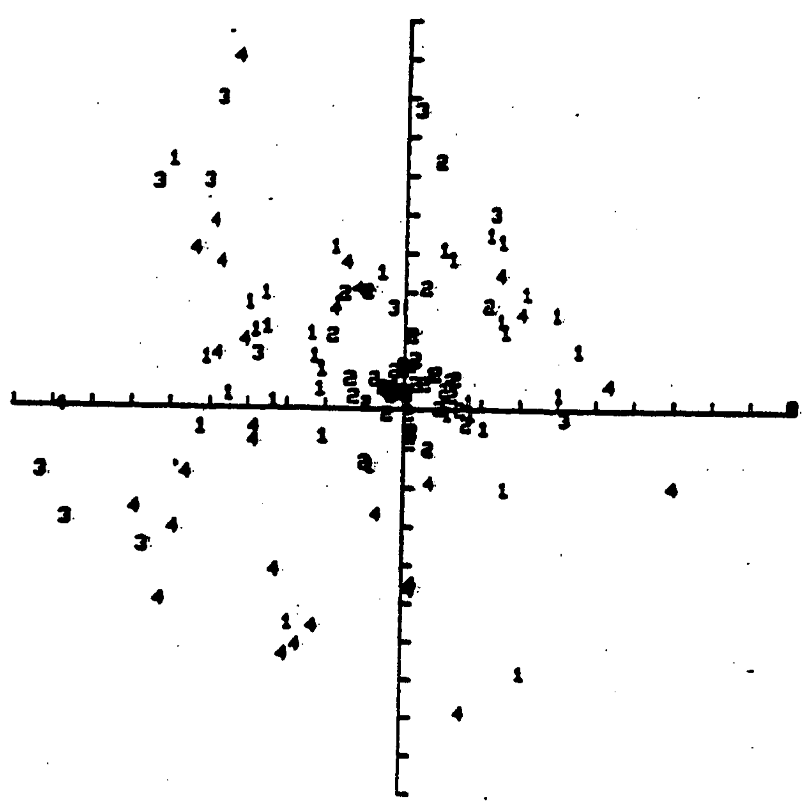

Figure 6.iiii : band D

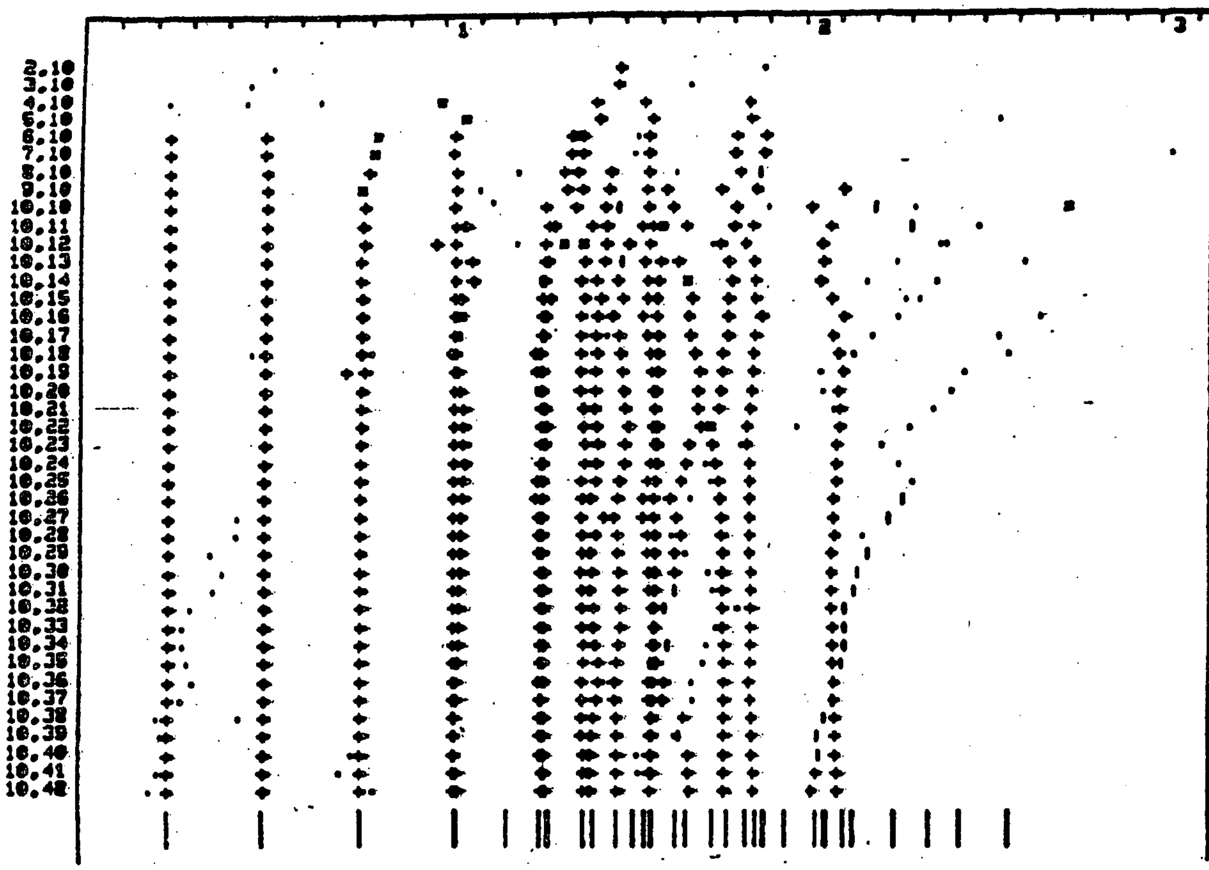

Figure 7.i : F.P.P., ARMA (10,10) model

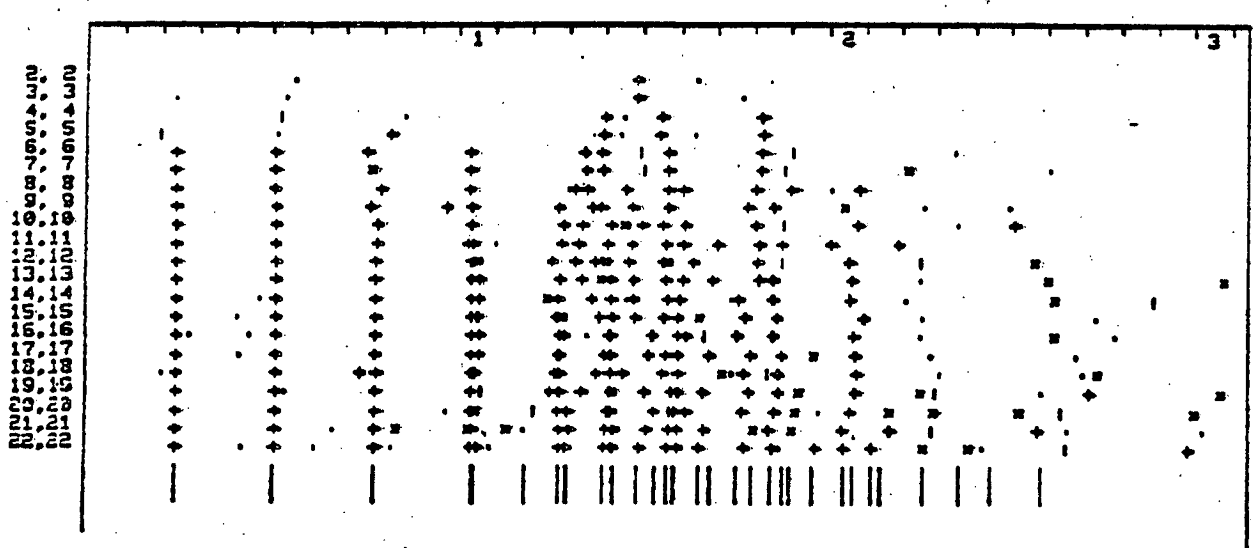

Figure 7.ii : I.V.; ARMA (n,n) model

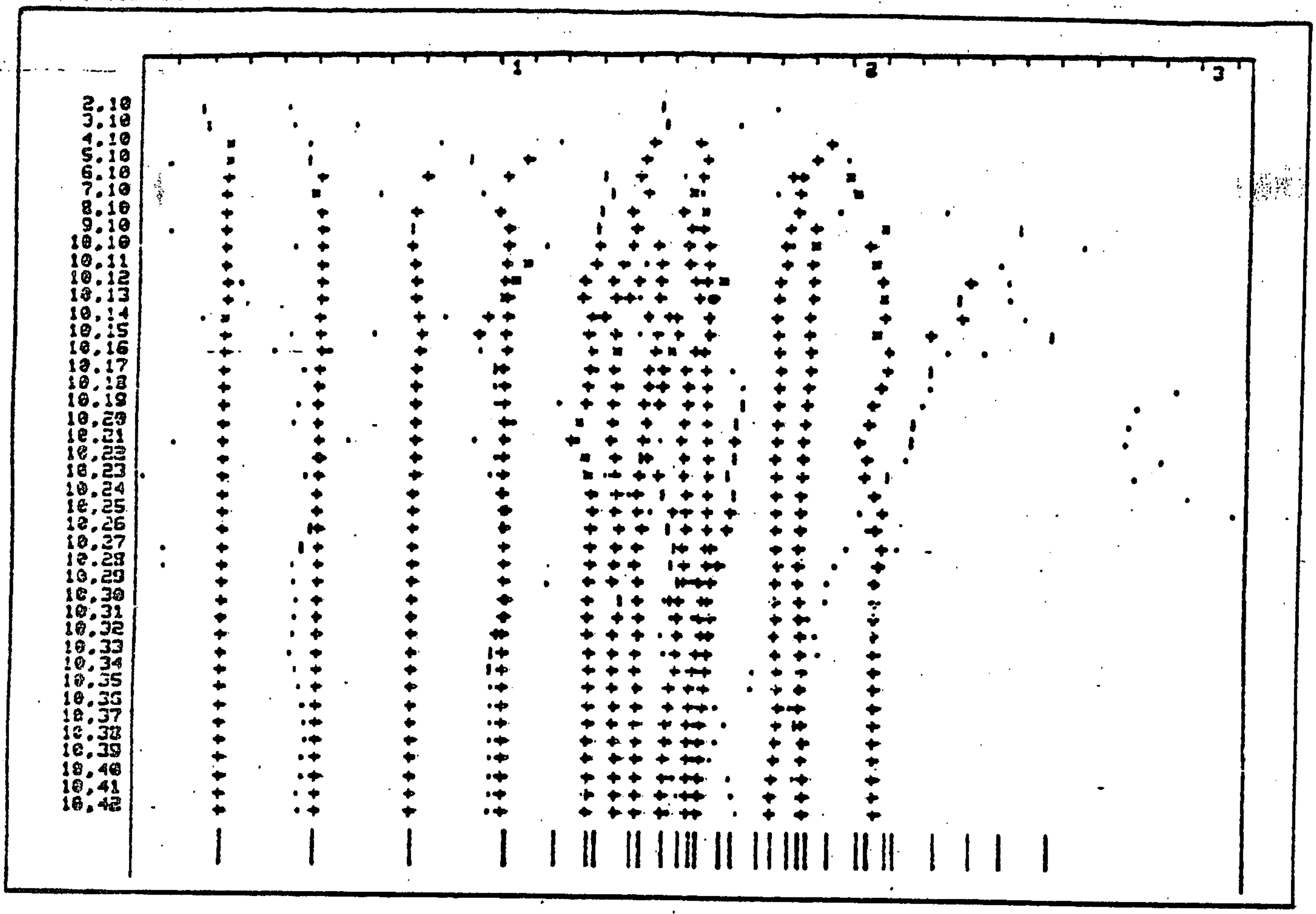

Figure 8.i : Stationary excitation, 3 masses excited, ARMA (10,10) model

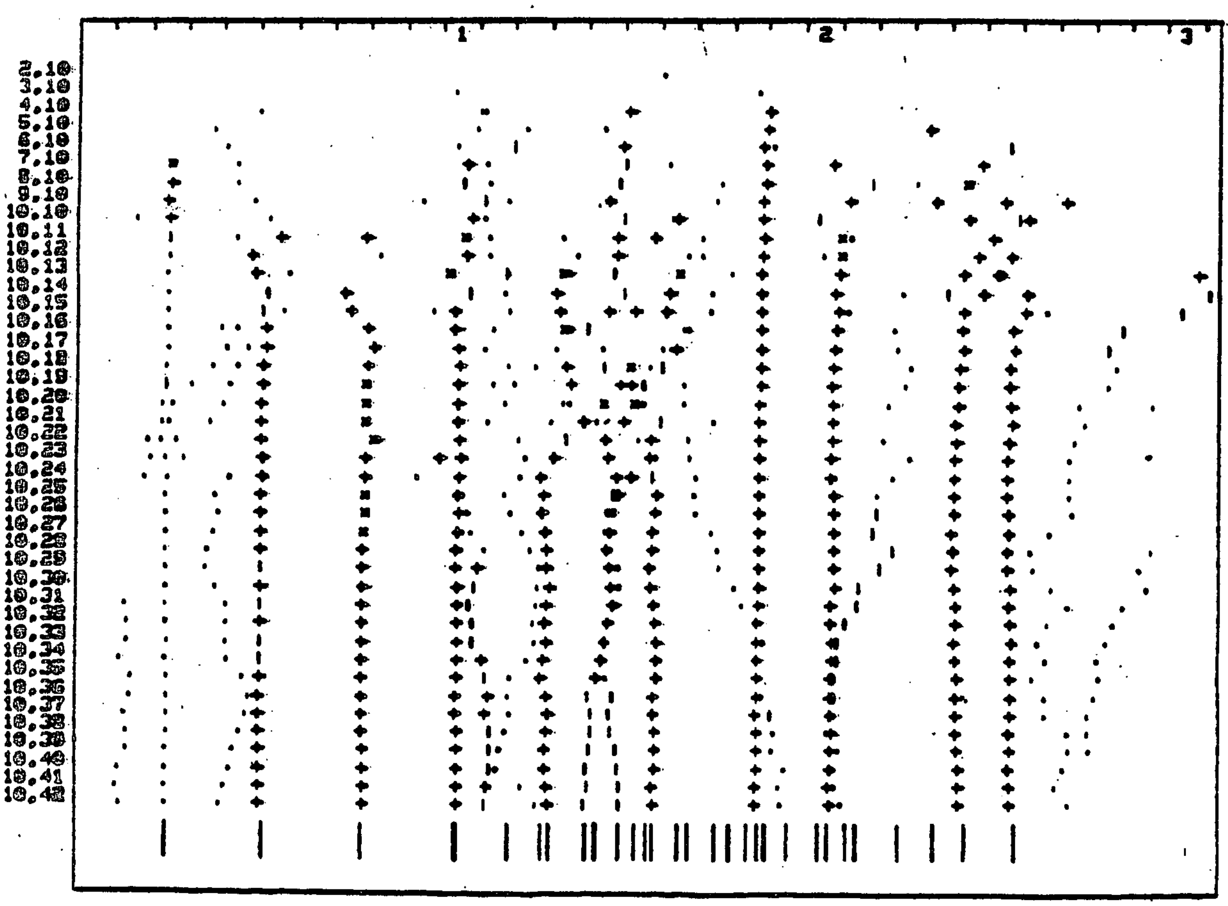

Figure 8.ii : Stationary excitations, all masses excited ARMA (10,10) model

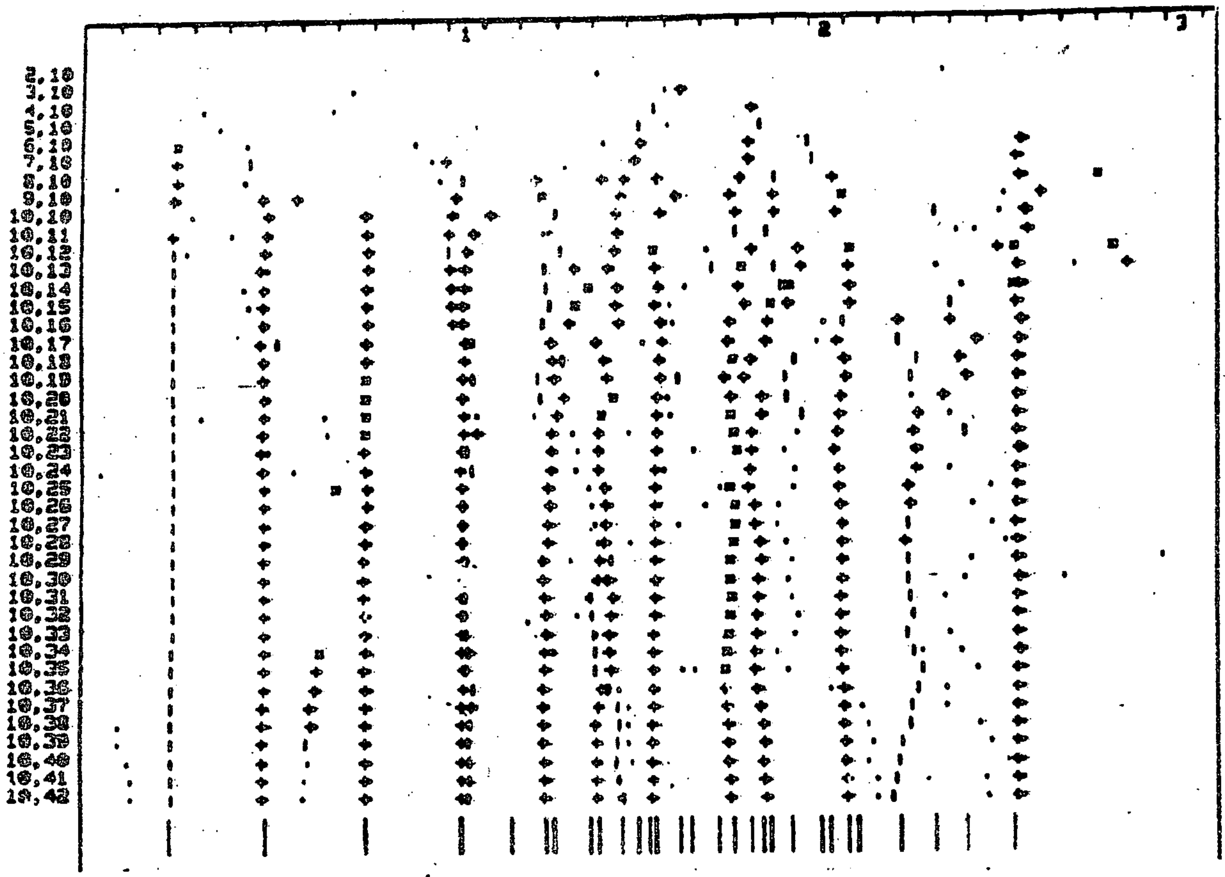

Figure 8.iii : non-stationary excitation, ARMA (10,10) model

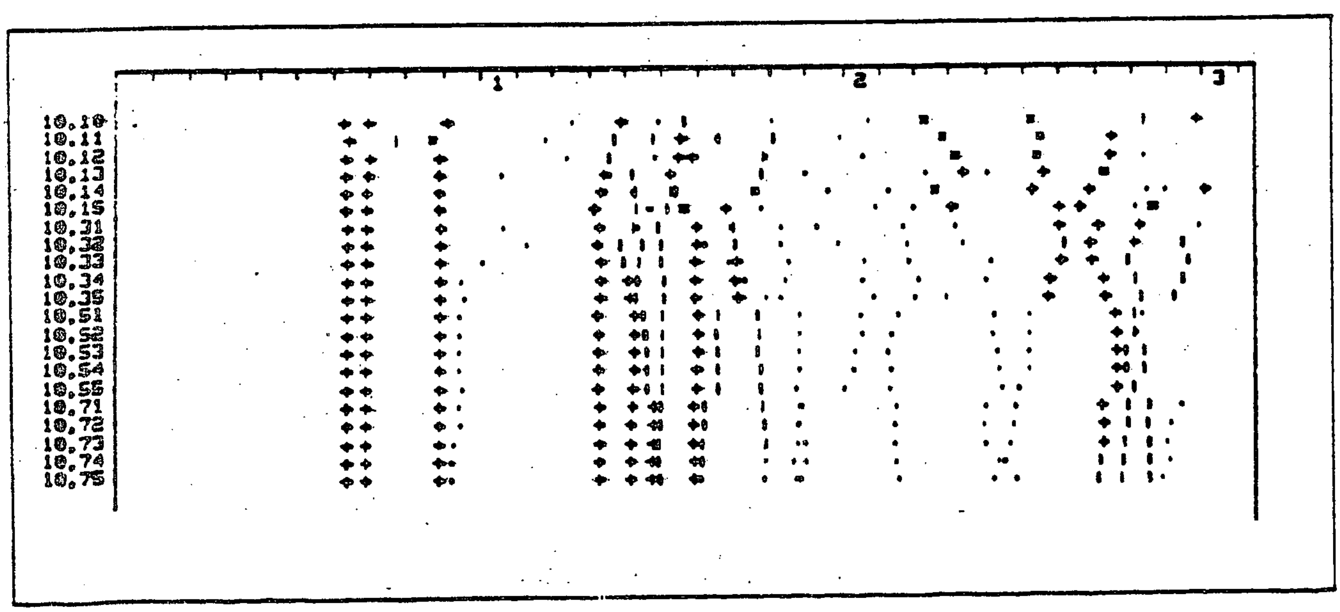

figure 9 : F.P.P., ARMA (10,10) model, 5800 samples

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